Group Project 2
Simpson's Rule
Due Wednesday, Jan. 27

*****Read this first!** – You will need to email your professor (me) to get the specific function and interval to use in your report. Be sure to do this early enough to get this data and complete your report. (e,g. if you email me at 11:00 PM, I probably will not get you the data until the following day.)

Project Assignment: Riemann sums can be used to approximate the value of an integral $\int_a^b f(x)\,dx$ but there are other methods that can produce better results with less work. In this project you will investigate another method for approximating the value of an integral called Simpson's rule. With Riemann sums, the function is approximated by a series of horizontal lines. The area under each line is calculated (the area of a rectangle) and the sum of these areas approximates the area under the original curve.

With Simpson's rule, the function is approximated with a polynomial of degree at most 2 (quadratic polynomial whose graph is a parabola). The area under the parabola is calculated and used to approximate the area under the original function.

1. First, use the fact that given 3 points in the plane with distinct x coordinates, there is a polynomial of degree at most 2 whose graph passes through all three points. Using the function obtained from your professor, write the equation of such a polynomial, $y = p(x)$, that passes through $(-1, f(-1))$, $(0, f(0))$, and $(1, f(1))$.

2. Graph f and p together over $[-1, 1]$.

3. Next compute
$$\int_{-1}^{1} p(x)\,dx$$
and compare the results to
$$\int_{-1}^{1} f(x)\,dx.$$

4. Let f be any continuous function and h a fixed positive number. Find a formula for the quadratic function p whose graph passes through the points $(-h, f(-h))$, $(0, f(0))$, and $(h, f(h))$. Find a formula for $p(x)$ in terms of the values of $f(-h)$,$f(0)$, and $f(h)$.

5. Show that
$$\int_{-h}^{h} p(x)\,dx = \frac{h}{3}[f(-h) + 4f(0) + f(h)].$$

6. Let c be a fixed number and let p be the quadratic function whose graph passes through the points $(c-h, f(c-h))$, $(c, f(c))$, $(c+h, f(c+h))$. Find a formula for $p(x)$ in terms of the values of $f(c-h)$, $f(c)$, and $f(c+h)$.

7. Evaluate
$$\int_{c-h}^{c+h} p(x)\, dx.$$

8. Your group is now ready to derive Simpson's rule. Let k be an integer and divide $[a, b]$ into $2k$ sections. Group them into k pairs of adjacent sections. Over each pair, estimate the integral of f as in step 7. Show that when these k separate estimates are added, Simpson's estimate
$$\int_a^b f(x)\, dx = \frac{h}{3}[f(a)+4f(a+h)+2f(a+2h)+4f(a+3h)+\cdots+2f(b-2h)+4f(b-h)+f(b)]$$
where
$$h = \frac{b-a}{2k}$$
is the result.

9. Use Simpson's rule with $k = 4$ to approximate the value of the integral for your particular function, $f(x)$, on the interval provided by your professor. Compare this to the exact value of the integral.

Your group should print your report and turn it in by the due date. Please be sure to fill out and return a project assessment form with your report.